



Physics of thermal waves in homogeneous and inhomogeneous (two-layer) samples

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Received 11 April 2001; accepted 13 February 2002

Abstract

Thermal waves produced by a periodic heat generation in homogeneous and inhomogeneous solids are examined from the theoretical point of view. The analysis is done for boundary conditions, thermal wave attenuation in a nondissipate medium and the physical meaning of “reflected thermal waves”. Separately it is discussed the comparison with electromagnetic waves. A new approach is suggested for calculation effective thermal conductivity and effective thermal diffusivity in two-layer structures within the frames of photothermal experiments. It is shown that the effective parameters depend on the physical properties of separate layers and interface, the manner of measuring these parameters, and the points of measuring.

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PACS: 65.90; 78.90

Keywords: Thermal waves; Photothermal experiments; Two-layer structure; Effective thermal conductivity; Effective thermal diffusivity

1. Introduction

A very active area of research in applied physics these days comes under the general heading of photothermal phenomena. Photothermal techniques in solid materials are becoming a valuable tool in measuring thermal parameters in solids and especially in the semiconductor industry for characterizing process in the manufacturing of electronic device [1]. These techniques are versatile, nondestructive and can be employed under different experimental conditions for determining thermal parameters of solids and liquid materials. Several photoacoustic cells with slight modifications, including the derivative photopyroelectric and photothermal deflection methods, have been used in some special cases with great success [2]. Recently, a new technique has been described in which a transient thermoelectric voltage of a semiconductor is measured after a pulse laser radiation [3].

In all the cases, the photothermal signal depends on the material thermal properties, interaction between the quasi-particle systems as well as on the geometry of the sample. The fact the photothermal signal depends upon how the heat diffuses through the sample, allows us to perform thermal characterization of the sample (i.e., measurements of its thermal conductivity and thermal diffusivity), and carrier transport properties.

The theoretical model of studying the thermal waves is based on the equation of heat flow. In the one-dimension case and in the absence of heat sources and sinks it is given as following [4],

$$\frac{\partial^2 T(x, t)}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T(x, t)}{\partial t} \quad (1)$$

where $T(x, t)$ is the temperature, $\alpha = \frac{\kappa}{\rho c}$ is the thermal diffusivity, κ is the heat conductivity, ρ is the density, c is the specific heat. The space coordinate is x , and t is time.

Many authors [5] propose an additional term $\tau_c \frac{\partial^2 T}{\partial t^2}$ in Eq. (1) where τ_c is the energy relaxation time. This new equation really represents a wave equation instead of a diffusion one and it is valid when $\tau_c > \omega^{-1}$, where ω being

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Nomenclature

α	thermal diffusivity	I_0	average over time light intensity
α_F	effective thermal diffusivity measured by the front surface illumination method	ΔI	modulating part of light intensity
α_R	effective thermal diffusivity measured by the rear surface illumination method	κ	thermal conductivity
d	sample's length	κ_F	effective thermal conductivity measured by the front surface illumination method
d_1	length of the first layer in the two-layer system	κ_R	effective thermal conductivity measured by the rear surface illumination method
c	specific heat	L	thermal diffusion length
c_l	light speed	ρ	density
E_i	component of the vector electric or magnetic field	Q_0	average over time of total heat flux at the surface
η	interface thermal conductivity	ΔQ	dynamic part of light-to-heat-conversion at the surface
$\tilde{\eta}$	surface thermal conductivity at the sample's surface $x = 0$	T	temperature
η_d	surface thermal conductivity at the sample's surface $x = d$	T_0	ambient temperature
		ω	modulation frequency

the frequency of the modulation light converted into heat in photothermal experiments [2]. However, it is well known that the temperature T is a thermodynamic parameter which describes the average energy of the system; thus if τ_c satisfy $\tau_c > \omega^{-1}$ then T is not a well defined parameter and Eq. (1) loses its physical meaning. In this work, we shall restrict ourselves to study conduction of heat in isotropic solids in which Eq. (1) is valid, i.e., $\tau_c \approx 0$.

Carslaw and Jaeger's [4] and other books contain a wealth of solutions of the heat-conduction equation. One of them usually corresponds to so-called thermal waves. A plane thermal wave has the form $e^{i\omega t + \sigma x}$, where $\sigma = (1 \pm i)(\frac{\omega}{2\alpha})^{1/2}$, $i = \sqrt{-1}$ (See Section 2). It is thus damped by a factor e^{-1} in a distance called by the thermal diffusion length $L = (\frac{2\alpha}{\omega})^{1/2}$.

Immediately a natural question arises: what does happen with the energy conservation in a nondissipate media where there are not any sources and sinks of heat? Besides, formally the term $e^{i\omega t + \sigma x}$ [4] represents a reflected thermal wave in the sample. If heat is a diffusion process: what is the physical interpretation of this term?

In the present work heat diffusion in one- and two-layer samples created by a periodic light beam is examined from the physical point of view. We restrict our analysis by solving the heat-diffusion equation assuming that the sample is optically opaque to the incident light. It is clear that when the intensity of the radiation is specified, the light-into-heat conversion at the surface $x = 0$ must be used as a boundary condition [6].

It is important to mention that the solutions of the heat diffusion equation obtained in this work in one-layer samples are not new, they were first obtained by Rosencwaig and Gersho [7] in the interpretation of the photoacoustic effect in solids on the basis of thermal waves. However, the

physical meaning of the solutions and the boundary conditions used in [7] are not completely clear.

Two-layer structures are often examined as a most simple model of multilayer, inhomogeneous structures under theoretical research. One of the main problems is to obtain expressions for the effective thermal parameters, being in agreement with the experimental measurements.

One of the popular approach is suggested by Mansanares et al. [8]. They used the analogy between thermal and electrical resistance in heat transfer problems. The effective thermal diffusivity obtained by these authors depends on relative thickness of materials, thermal diffusivities of each layer and the ratio of thermal conductivities of materials. Recently, Lucio et al. [9] have generalized the "resistance analogy" model. They have showed that equations arisen from "thermal-resistance analogy" model are valid only when both materials are thermally thin (thermal diffusion length is much more than the each layer's length). In the case of thermally thick components (thermal diffusion length is much less than the layer's length), the equation for the effective thermal diffusivity was obtained, and it was shown that it depends not only upon the thermal parameters of constituents but on the chopping frequency too.

About the last mentioned works (see also [10]) we have our opinion. The "electric and thermal analogy" is not adequate to describe a photothermal experimental situation. Potential difference only has the physical sense in electrodynamics. In nonequilibrium thermodynamics, not only temperature difference has physical sense (the analogy with potential difference) but the temperature itself too; it can be measured at Kelvin absolute scale. Just the temperature or some response to this temperature, is measured in photothermal experiments, but not the temperature difference [11].

We suggest another approach for calculation of effective thermal conductivity and thermal diffusivity of two-layer

samples. Every instrument measures temperature or temperature response in photoacoustic experiments only at one of the surface of the sample, the volume is the “black body” for it. Therefore the main idea comes to an imaginary replacement of the real two-layer structure by an effective one-layer sample. The demand of equality of boundary conditions on the left or right surface (where the measurement is carried out) is necessary. Thermal parameters of effective sample are the effective parameters of the real two-layer structure if the temperature at chosen surface of these samples is the same.

It is important to emphasize that equalization of temperatures on either left or right sides of the real two-layer system and the effective one-layer sample (but not temperature difference) can provide the different expressions for effective parameters. It means that the effective thermal conductivity and thermal diffusivity, in general, depend on how we carry out the measurements and the points of these measurements.

Generally, two different ways of the sample heating are used in photoacoustic experiments being the most popular of photothermal technique [12,13]. First of them is the front surface illumination (the detecting photoacoustic cell is replaced at the irradiated sample surface, and it is called by the close photoacoustic cell). The second one is the rear surface illumination (the photoacoustic cell is replaced at the back sample surface, and it is called by the open photoacoustic cell). In the first case the nonequilibrium temperature is detected at the same surface on which the modulated laser beam falls. The second method assumes the detection this temperature at the opposite sample side.

With respect to photoacoustic experiments it means that we can measure different values of effective thermal conductivity and effective thermal diffusivity depending on which illumination method is utilized, the closed acoustic cell or the open one.

2. One-layer sample. Boundary conditions

In this paper we are considering that the thermal conductivity and the thermal diffusivity are independent on the nonequilibrium temperature, so this approximation is valid when the intensity of the incident radiation is not strong. The solution $T(x, t)$ should be supplemented by the boundary conditions at the sample surfaces $x = 0$ and $x = d$, where $x = d$ is the sample surface opposite to the irradiated surface $x = 0$. In the photothermal experiments, the common mechanism to produce thermal waves is the absorption an incident modulated light beam $I_0 + \Delta I e^{i\omega t}$ at the surface $x = 0$, where I_0 is the average over time the light intensity, ΔI is the modulating part of light intensity, ω is the modulation frequency. The lateral sides of the sample are supposed to be adiabatically insulated.

In the case of surface absorption one can write the boundary condition at the surface $x = 0$ as follows:

$$-\kappa \frac{\partial T}{\partial x} \Big|_{x=0} = [(Q_0 + \Delta Q e^{i\omega t}) - \tilde{\eta}(T - T_0)]_{x=0} \quad (2)$$

Here Q_0 is the average over time of the total heat flux $Q(x, t)$ at the surface of the sample, and it is proportional to the intensity I_0 . Physically, this static heat flux at the surface gives rise to a non-depending on time nonequilibrium temperature distribution in the specimen. The term ΔQ corresponds to the dynamic part of light-to heat-conversion at the surface, and it is proportional to the intensity ΔI . The ambient temperature is T_0 . This surface heat source has sinusoidal time dependence and it produces thermal wave propagation into the sample. The second term in the square bracket in Eq. (2) represents the heat flux from the surface of the sample to the surrounding media. At the limit when surface thermal conductivity $\tilde{\eta} \rightarrow 0$ (adiabatic approximation), the heat boundary condition is only given by the first term. It is necessary to notice that both Q_0 and ΔQ are phenomenological parameters in the present paper.

The temperature itself cannot be used as the boundary condition in the theoretical study of the photothermal experiments because it is usually an unknown parameter. It can be used as the boundary condition only if the sample is heated by the contacting heater with the given temperature.

In this connection, it is important to compare the boundary conditions used in this work with the other ones used in previous theories. Rosencwaig and Gersho [7] and many other scientists consider the temperature and heat flow continuity at the surface $x = 0$ in order to describe thermal waves in photothermal experiments. Opsal and Rosencwaig [14], on the other hand, used a periodic heat source $Q e^{i\omega t}$ at the surface of a semi-infinite body as boundary condition to study the thermal wave depth profiling. From our point of view, the correct equation for the heat flux at the surface must be written by $Q(x = 0, t) = Q_0 + \Delta Q e^{i\omega t}$, where $Q_0, \Delta Q > 0$; $\Delta Q e^{i\omega t}$ can be positive or negative at different moment of time, and $\Delta Q < Q_0$. In according with Opsal and Rosencwaig model, only the periodic surface heat source generates heat diffusion process, the contribution of the static light intensity is ignored. In this case the amplitude of the modulation heat is greater than average over the time value $\langle Q e^{i\omega t} \rangle = 0$, and can take negative values. It is clear that this situation has no physical sense.

The boundary condition at the surface $x = d$ can be taken in the form

$$-\kappa \frac{\partial T}{\partial x} \Big|_{x=d} = \eta_d (T - T_0) \quad (3)$$

Nevertheless, for simplicity, we suppose that the thermal contact at this surface is perfect ($\eta_d \rightarrow \infty$), so we can write

$$T(x, t)|_{x=d} = T_0 \quad (4)$$

3. Temperature distribution in one-layer sample

Let us write the general solution of the heat diffusion equation (1) in the form

$$T(x, t) = A + Bx + \Theta(x)e^{i\omega t} \quad (5)$$

where A , B are unknown constants, $\Theta(x)$ is unknown function, determining the coordinate dependence of the thermal wave and satisfying the equation

$$\frac{d^2\Theta}{dx^2} = \frac{i\omega}{\alpha} \Theta \quad (6)$$

Supposing for simplicity that $\tilde{\eta} = 0$ at the surface $x = 0$ and $\eta_d = \infty$ at the surface $x = d$ the boundary conditions take the form:

$$\begin{aligned} -\kappa \frac{d\Theta(x)}{dx} \Big|_{x=0} &= \Delta Q \\ \Theta(x) \Big|_{x=d} &= 0 \end{aligned} \quad (7)$$

In this case the constants A , B are the following,

$$A = T_0 + \frac{Q_0}{\kappa}d, \quad B = -\frac{Q_0}{\kappa} \quad (8)$$

Thus, the solution of Eq. (6) is

$$\Theta(x) = f_1 e^{-\sigma x} + f_2 e^{\sigma x} \quad (9)$$

where

$$f_1 = \frac{e^{\sigma d}}{e^{\sigma d} + e^{-\sigma d}} \frac{\Delta Q}{\kappa \sigma}, \quad f_2 = \frac{e^{-\sigma d}}{e^{\sigma d} + e^{-\sigma d}} \frac{\Delta Q}{\kappa \sigma} \quad (10)$$

and $\sigma = (1 + i)\sqrt{\frac{\omega}{2\alpha}}$.

Combining Eqs. (5) and (8)–(10) we obtain

$$T(x, t) = T_0 + \frac{Q_0}{\kappa}(d - x) + \frac{\Delta Q}{\kappa} \frac{\sinh \sigma(d - x)}{\cosh \sigma d} e^{i\omega t} \quad (11)$$

It is follows from Eq. (11) that the dynamic temperature distribution depended on time attenuates to zero with increasing distance from the surface $x = 0$. At the distance $L \approx |\sigma|^{-1} = \sqrt{\frac{2\alpha}{\omega}} = \sqrt{\frac{2\kappa}{\rho c \omega}}$ this contribution to the temperature fluctuation is effectively damped out. Besides, there is the “reflected” thermal wave. Both these facts are needing in the explanation because we are examining the heat diffusion process.

For the electromagnetic waves the attenuation is associated with the energy transfer to the charge particles setting in the motion.

For the thermal waves the reason of attenuation is quite different. Let us represent the elementary calculations which will help to understand the essence of the matter.

Let us write the temperature (11) in the form,

$$T(x, t) = T^s(x) + T^d(x, t)$$

Here $T^s(x) = T_0 + \frac{Q_0}{\kappa}(d - x)$ is the static part of the temperature distribution,

$$T^d(x, t) = \frac{\Delta Q}{\kappa} \frac{\sinh \sigma(d - x)}{\cosh \sigma d} e^{i\omega t} \quad \text{is the dynamic part.}$$

The heat quantity evolving or absorbing in the unite volume and per unite time is equal to $\Delta Q = -\text{div} \vec{q}$, where \vec{q} is the heat flux density. In our case $\vec{q} = \vec{q}^s + \vec{q}^d$, where in one-dimensional case $q^s = -\kappa \frac{dT^s}{dx}$ is the density of the static heat flux, and $q^d = -\kappa \frac{dT^d}{dx}$ is the density of the dynamic heat flux.

It easy to see, that $\Delta Q^s = -\text{div} \vec{q}^s = 0$, i.e., the static heat flux is not absorbed. At the same time $\Delta Q^d = -\text{div} \vec{q}^d = \rho c \frac{\partial T^d}{\partial t} \neq 0$. From this result it is follows that the attenuation of the thermal wave is caused by the presence of the heat capacity and the temperature variation on time. This variation is proportional to the modulation frequency. Thus, the matter’s property to accumulate heat energy is the main reason of the thermal wave damping. What why the decay distance L rapidly decreases with increasing of the heat capacity and the modulation frequency.

The another question is: what is the physical meaning of the “reflected heat wave”? The answer is the following: The modulation part of the dynamic heat flux $\Delta Q e^{i\omega t}$ at the irradiated surface can be positive or negative. So during a half period of the modulation incident radiation, i.e., for $\frac{\pi}{\omega}$, the dynamic part of the temperature at this surface is higher than inside of the sample and during this time there is propagation of heat from the surface to the bulk of solid. This effect is associated with the decreasing exponential term in Eqs. (9) and (11). On the other hand, the growing exponential term in these Eqs. represents the heat flux from inside to the surface of the sample when $\Delta Q e^{i\omega t}$ is negative, i.e., now, during the period of time $\frac{\pi}{\omega}$ the dynamic part of the temperature in the bulk is higher than at the surface.

As can be seen from Eq. (11), the dynamic contribution to the temperature distribution has a sinusoidal dependence through the imaginary part of the exponential terms, i.e., $e^{\pm i\sqrt{\frac{2\alpha}{\omega}}x}$. Nevertheless, the propagation of the thermal waves in the sample is not similar to the propagation of the electromagnetic waves having formally the same shape. The heat flux is a diffusion process, and it is described by the heat diffusion equation (1), while the propagation of electromagnetic waves satisfies the wave equation.

4. Comparison with electromagnetic waves

We believe, at this point, that it is important to emphasize the main differences between the propagation of electromagnetic waves (can also be sound waves) and thermal waves. The main of them is the following: The propagation of the plane transverse electromagnetic waves satisfies the wave equation

$$\frac{\partial^2 E_i}{\partial x^2} = \frac{\varepsilon}{c_i^2} \frac{\partial^2 E_i}{\partial t^2} \quad (12)$$

which is a hyperbolic equation, while the temperature satisfies a parabolic equation (see Eq. (1)). Here E_i ($i = x, y, z$) is a component of the vector electric or magnetic

field, c_l is the light speed in the matter, ε is the complex dielectric constant [15].

The any known wave (sound, electromagnetic) is transverse or longitudinal, besides transverse waves are polarized. These concepts are absent for the thermal waves.

In general, the solution of the wave equation for the electromagnetic field is

$$E_i(x, t) = E_{i1}e^{-\lambda x}e^{i(\omega t - kx)} + E_{i2}e^{\lambda x}e^{i(\omega t + kx)} \quad (13)$$

where $E_{i1,2}$ are constants, λ is the light absorption coefficient, k is the wave number [15].

On the other hand, the time-dependence solution of the heat diffusion equation, Eq. (11), is given by

$$T(x, t) = T_1e^{-\gamma x}e^{i(\omega t - \gamma x)} - T_2e^{\gamma x}e^{i(\omega t + \gamma x)} \quad (14)$$

where $T_{1,2}$ are complex values, and $\gamma = L^{-1} = \sqrt{\frac{\omega}{2\alpha}}$.

Although, Eqs. (13), (14) are mathematically similar, the physical meaning of them are different.

Firstly, it is the change on sign in the solution of the diffusion equation, Eq. (13) as compared with Eq. (14). In the wave problem the total flux density of energy in the sample is the difference of the flux density energy of both, the transmitted and reflected waves and it is proportional to $|E_i|^2$ (Poynting vector). In the heat diffusion process it is the sum. It corresponds to the rate at which the heat is generated or absorbed at various places in the sample and because the energy heat flux can be positive or negative

$$\Delta Q(x, t) = -\kappa \frac{\partial T(x, t)}{\partial x}$$

Secondly, it is important to note that the following inequality $k \geq \lambda$ is always fulfilled in the solution of the wave problem [15]. The absorption coefficient and the wave number of the thermal waves are the same (see Eq. (14)). The reason of this is the complex value of the right-hand of Eq. (12) while the right-hand of Eq. (6) is the pure imaginary quantity $\frac{i\omega}{\alpha}$. All these differences mentioned in the text show that the analogy between thermal “waves” and electromagnetic waves is not correct.

5. Temperature distributions in two-layer structure, and equations for effective thermal parameters

Similarly to the previous sections let us assume that the energy of the modulated laser beam is converted into heat completely onto the surface $x = 0$ of the two-layer sample (see Fig. 1). Each layer is homogeneous and isotropic and the cross section at any plane perpendicular to the axis ox is equal to unity. The opposite surface $x = d$ is in contact with a thermostat at a temperature T_0 ; the lateral sides are adiabatically insulated. Thus the problem is one-dimensional and the temperature distributions in two-layer structure can be obtained from the following heat diffusion equations:

$$\frac{\partial^2 T_{1,2}(x, t)}{\partial x^2} = \frac{1}{\alpha_{1,2}} \frac{\partial T_{1,2}(x, t)}{\partial t} \quad (15)$$

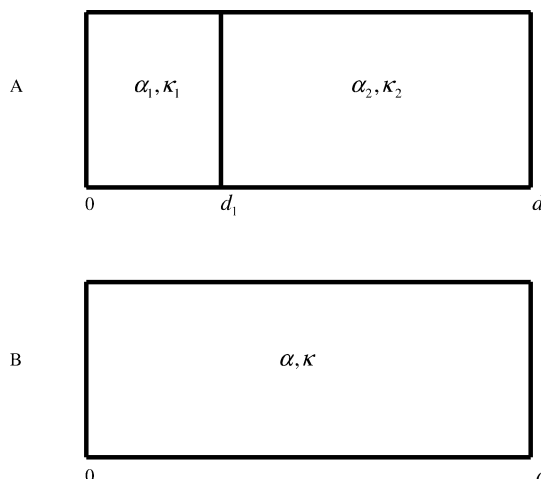


Fig. 1. (A) Two-layer structure; (B) One-layer effective sample.

where $T_{1,2}(x, t)$ are the temperatures, $\alpha_{1,2} = \kappa_{1,2}/(\rho c)_{1,2}$ are the thermal diffusivities, $\kappa_{1,2}$ are the thermal conductivities, $\rho_{1,2}$ are the densities, and $c_{1,2}$ are the specific heats in the first and the second layer, respectively.

In this problem we take into account the thermal properties of interfacial region between layers. Really it has some thickness and some thermal conductivity different from the thermal conductivities in the principal volumes of the layers. In the model of surface interface its thickness tends to zero and the heat property of this region is described by the some surface thermal conductivity η . The temperature gap takes place at the interface plane $x = d_1$ when the thermal conductivity η has the finite magnitude [16]. In this case within the interface region the thermal flux is defined by the following temperature difference $T_2(x = d_1 + 0) - T_1(x = d_1 - 0)$. Simultaneously the condition of continuity of thermal fluxes must takes place at the plane $x = d_1$.

Thus, the boundary conditions to Eq. (15) can be written in the following form (compare with [6]),

$$-\kappa_1 \frac{\partial T_1(x, t)}{\partial x} \Big|_{x=0} = Q_0 + \Delta Q e^{i\omega t} \Big|_{x=0} \quad (16.1)$$

$$\kappa_1 \frac{\partial T_1(x, t)}{\partial x} \Big|_{x=d_1} = \kappa_2 \frac{\partial T_2(x, t)}{\partial x} \Big|_{x=d_1} \quad (16.2)$$

$$-\kappa_1 \frac{\partial T_1(x, t)}{\partial x} \Big|_{x=d_1} = \eta(T_1(x, t) - T_2(x, t)) \Big|_{x=d_1} \quad (16.3)$$

$$T_2(x, t) \Big|_{x=d} = T_0 \quad (16.4)$$

The solutions of Eq. (15) with the boundary conditions (16) are as follows,

At the interval $0 \leq x \leq d_1$,

$$T_1(x, t) = A_1 + B_1 x + \{ U_1 \cosh[\sigma_1(x - d_1)] - U_2 \sinh[\sigma_1(x - d_1)] \} e^{i\omega t} \quad (17)$$

At the interval $d_1 \leq x \leq d$,

$$T_2(x, t) = A_2 + B_2(x - d_1) + \left\{ \frac{2(\Delta Q / (\kappa_2 \sigma_2)) \sinh[\sigma_2(d - x)]}{D} \right\} e^{i\omega t} \quad (18)$$

where

$$\begin{aligned}
 U_1 &= \frac{2(\Delta Q/(\kappa_1\sigma_1))\left[\frac{\kappa_1\sigma_1}{\kappa_2\sigma_2} \sinh(\sigma_2 d_2) + \frac{\kappa_1\sigma_1}{\eta} \cosh(\sigma_2 d_2)\right]}{D} \\
 U_2 &= \frac{2(\Delta Q/(\kappa_1\sigma_1)) \cosh(\sigma_2 d_2)}{D} \\
 A_1 &= T_0 + \frac{Q_0}{\eta} + Q_0 \left(\frac{d_1}{\kappa_1} + \frac{d_2}{\kappa_2} \right) \\
 A_2 &= T_0 + \frac{Q_0}{\kappa_2} d_2 \\
 B_1 &= -\frac{Q_0}{\kappa_1} \\
 B_2 &= -\frac{Q_0}{\kappa_2} \\
 \sigma_{1,2} &= (1+i)q_{1,2} \\
 q_{1,2} &= \sqrt{\frac{\omega}{2\alpha_{1,2}}} \\
 D &= \left(1 - \frac{\kappa_1\sigma_1}{\kappa_2\sigma_2}\right) \cosh(\sigma_1 d_1 - \sigma_2 d_2) \\
 &\quad + \left(1 + \frac{\kappa_1\sigma_1}{\kappa_2\sigma_2}\right) \cosh(\sigma_1 d_1 + \sigma_2 d_2) \\
 &\quad + \left(\frac{\kappa_1\sigma_1}{\eta}\right) [\sinh(\sigma_1 d_1 - \sigma_2 d_2) + \sinh(\sigma_1 d_1 + \sigma_2 d_2)]
 \end{aligned} \tag{19}$$

It is easy to see that the temperature of each layer depends on the thermal bulk parameters of both layers and on the interface thermal conductivity too. It is obviously that the photothermal signal includes itself an information about the integral thermal conductivity and thermal diffusivity which are the function of all parameters of both layers. What are these functions is the main question of the examined problem.

Let us now imagine some homogeneous one-layer sample having the same geometric shape like the real two-layer system with thermal conductivity κ , thermal diffusivity α and the thickness $d = d_1 + d_2$ (see Fig. 1).

Let us call the values κ_F and α_F the effective thermal conductivity and effective thermal diffusivity measured by the front surface illumination method (FSIM) if they are obtained from the following equation,

$$T(x=0, t) = T_1(x=0, t) \tag{20}$$

Similarly one could define the effective thermal conductivity κ_R and the effective thermal diffusivity α_R under the measurement by the rear surface illumination method (RISM), i.e., from the side $x = d$. If contact $x = d$ is isothermal we cannot compare temperatures $T_2(x)$ and $T(x)$ on the surface $x = d$ because here they are equal to the ambient temperature (see boundary conditions (16,4) and (4)). So the procedure of obtaining thermal parameters by RISM relates to the points closely located to $x = d$, i.e., in the point $x = d - \delta$. Here δ is the infinitely small distance. In this point both the real and effective temperature are nonequilibrium.

In this case

$$T(x = d - \delta, t) = T_2(x = d - \delta, t) \tag{21}$$

is the equation to calculate κ_R and α_R by RISM.

We postulate that the values of all this effective parameters are adequate to the experimental situation.

6. Effective thermal parameters

Eqs. (20) and (21) in general case are very complicated, complex, and transcendental equations for obtaining both, effective thermal conductivity and effective thermal diffusivity. Therefore, for simplicity let us discuss some special cases corresponding to low modulation frequencies. Since the frequency $2\alpha_i/d_i^2$ is the characteristic frequency of i th layer, then the criterions of low frequencies is the following inequalities:

$$\omega \ll \frac{2\alpha_i}{d_i^2} \quad \text{or} \quad d_i \ll L_i = |\sigma_i|^{-1} \tag{22}$$

where

$$|\sigma_i|^{-1} = \sqrt{\frac{2\alpha_i}{\omega}} \tag{22'}$$

The latter inequalities determine so-called thermally thin samples.

It is clear that the modulation frequency ω can be in arbitrary relations with the characteristic frequencies $\omega_i = \frac{2\alpha_i}{d_i^2}$ corresponding to the different layers because ω is the independent value.

Thus, the front surface illumination measurements (FSIM) (Eq. (20)), results in the following effective parameters,

$$\kappa_F = \frac{d_1 + d_2}{d_1/\kappa_1 + d_2/\kappa_2 + 1/\eta} \tag{23}$$

$$\alpha_F = \frac{(d_1 + d_2)^3}{\kappa_F(d_1^3/\alpha_1\kappa_1 + d_2^3/\alpha_2\kappa_2) + 3\frac{\kappa_1 d_1 d_2 (d_1 + d_2)}{\alpha_1 \kappa_2} (1 + \frac{\kappa_2}{\eta d_2})} \tag{24}$$

Under the rear surface illumination measurements (RISM) (see Eq. (21)), the effective parameters are another,

$$\kappa_R = \kappa_2 \tag{25}$$

$$\alpha_R = \frac{d^2}{d_1^2/\alpha_1 + d_2^2/\alpha_2 + 2(\kappa_1/\kappa_2)(d_1 d_2/\alpha_1)(1 + \kappa_2/(\eta d_2))} \tag{26}$$

Thus, we come to the very important result, that the effective parameters measured by the photothermal techniques are not the one-valid parameters. Their magnitudes depend on the manner of measurement and the point of measurement.

The effective thermal diffusivity depends on the all heat parameters of the two-layer system such as the thermal conductivity κ_1 , κ_2 ; thermal diffusivities α_1 , α_2 ; layer

geometric sizes and the interface thermal conductivity η . At the same time the effective thermal conductivity does not depend on the thermal diffusivities.

It is clear that the interface does not influence on the effective parameters in the case of isothermal contact between the layers. It follows from Eqs. (23), (24) and (26) that the criterion of this isothermally are different for the effective thermal conductivity and effective thermal diffusivity. This criterion is

$$\eta \gg \frac{\kappa_1 \kappa_2}{\kappa_2 d_1 + \kappa_1 d_2} \quad (27)$$

for the effective thermal conductivity. It includes itself the thermal conductivities of both layers and its thickness.

At the same time

$$\eta \gg \frac{\kappa_2}{d_2} \quad (28)$$

is the criterion of isothermally for the effective thermal diffusivity. It is determined by the correlation between the interface thermal conductivity, the heat conductivity of the second layer and its thickness.

It is interesting to examine separately the influence of the interface on the effective parameters values. Let us assume that the two-layer system consists of two identical layers ($d_1 = d_2$, $\kappa_1 = \kappa_2$, $\alpha_1 = \alpha_2$) divided by some interface having the finite surface thermal conductivity. The measurements by the method of closed or open cells result in the different heat effective parameters even in this elementary case. Really, under front illumination we have:

$$\kappa_F = \kappa_1 \frac{1}{1 + \kappa_1 / (2d_1 \eta)} \quad (29)$$

$$\alpha_F = \alpha_1 \frac{4}{3(1 + \kappa_1 / (d_1 \eta)) + (1 + \kappa_1 / (2d_1 \eta))^{-1}} \quad (30)$$

Here κ_1 , α_1 , d_1 are the thermal conductivity, thermal diffusivity and thickness of layer, respectively.

The same parameters obtained by the method of open cell are following,

$$\kappa_R = \kappa_1 \quad \text{and} \quad \alpha_R = \alpha_1 \frac{1}{1 + \frac{\kappa_1}{2d_1 \eta}} \quad (31)$$

The measurements by closed and open cells lead to the same results only in the case of isothermal contact, but it is just the one-layer sample.

7. Conclusions

In conclusion, a theoretical analysis of thermal waves has been studied. Using the appropriate boundary conditions adequate to the photothermal experiments, we have obtained the temperature distributions in one and two-layer samples. It is examined the physical reason of the thermal wave attenuation and the meaning of the “reflected thermal wave”.

We have suggested a new approach to obtain the effective thermal conductivity and the effective thermal diffusivity of one-dimensional two-layer structures when the photothermal methods are used. One of the principal statements tells that under different photothermal measurements (close cell or open cell) we obtain different values of the thermal parameters occur in general case. Besides, the finite values of the interface thermal conductivity leads to essential modification of both the effective thermal conductivity and the effective thermal diffusivity.

Acknowledgements

This work has been partially supported by Consejo Nacional de Ciencia y Tecnología (CONACYT), México.

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